L.S.T.A.
Laboratoire de Statistique Théorique et Appliquée

Mathematical Statistics and Limit Theorems

Conference in honor of Prof. Paul Deheuvels

June 20-21th 2013 Université Pierre et Marie Curie Paris VI

Organizing committee

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Programme

Jeudi 20 juin 2013

8.30 - 9.00 : Accueil des participants et des invités
9.00 - 9.15 : Mot d’ouverture Madan L. Puri (Indiana University Bloomington)

Session I : Chairman : Erich Haeusler

9.15 - 10.00 : Conférence plénière David Mason (University of Delaware)
  Strong approximations to a quantile process based on independent fractional
  Brownian motions

10.00 - 10.25 : Philippe Berthet (Université de Toulouse 3)
  Limit Theorems for Spatial Quantiles
  Pause café

Session II : Chairman : Philippe Berthet

11.00 - 11.25 : Erich Haeusler (Universität Giessen)
  The sequential empirical process for centered random variables

11.25 - 11.50 : Holger Rootzén (Chalmers University Göteborg)
  Error distributions for random grid approximations of
  multidimensional stochastic integrals

11.50 - 12.15 : Victor de la Peña (Columbia University)
  From Boundary Crossing of Non-Random Functions to Boundary
  Crossing of Stochastic Processes
  Pause déjeuner

Session III : Chairman : David Mason

14.00 - 14.45 : Conférence plénière Dietmar Pfeifer (Universität Oldenburg)
  Semigroups and Probability: From representation theorems to
  Poisson approximation

14.45 - 15.10 : Mikhail A. Lifshits (University of Saint Petersburg and Linköping University)
  Approximation of Processes and Operators of Variable Non-Smoothness

15.10 - 15.35 : István Berkes (Universität Graz)
  Weakly dependent processes in analysis
  Pause café

Session IV : Chairman : Josef G. Steinebach
16.00 - 16.25 : John Einmahl (Tilburg University)  
*Bridging centrality and extremity: refining empirical data depth using extreme value theory*

16.25 - 16.50 : Armelle Guillou (Université de Strasbourg)  
*Reduced-biased estimators of the Conditional Tail Expectation for heavy-tailed distributions*

16.50 - 17.15 : Rolf-Dieter Reiss (Universität Siegen)  
*Asymptotic Expansions of Distributions of Multivariate Maxima*

17.15 - 17.40 : Jan Beirlant (Katholieke Universiteit Leuven)  
*An Overview and Open Research Topics in Statistics of Univariate Extremes*

**Vendredi 21 juin 2013**

**Session I : Chairman: Zhan Shi**

9.00 - 9.15 : Mot d’ouverture, George G. Roussas (University of California, Davis, USA)

9.15 - 10.00 : Conférence plénière Joseph Steinebach (Universität zu Köln)  
*Erdős-Rényi-Shepp Type Laws and Some Related Phenomena*

10.00 - 10.25 : Uwe Einmahl (Vrije Universiteit Brussel)  
*Strassen type LIL results for partial sum processes in Euclidean space*

**Pause café**

**Session II : Chairman Gérard Biau**

11.00 - 11.25 : Denis Bosq (Université Paris VI)  
*Bayesian prediction for stochastic processes*

11.25 - 11.50 : Adrian Raftery (University of Washington)  
*Bayesian Reconstruction of Past Populations for Developing and Developed Countries*

11.50 - 12.15 : Javier Girón (University of Malaga)  
*Asymptotics of Bayesian variable selection procedures in linear regression*

**Pause déjeuner**

**Session III : Chairman George G. Roussas**

14.00 - 14.45 : Conférence plénière Marc Hallin (Université libre de Bruxelles)  
*Signal Detection in High Dimension: testing sphericity against spiked alternatives*

14.45 - 15.10 : Mikhaïl Nikulin (Université de Bordeaux-2)  
*Recent contributions to the theory of chi-squared tests*

15.10 - 15.35 : Yacov Yu. Nikitin (University of Saint Petersburg)  
*Limit Theorems for Perimeters and Areas of Random Polygons*
Pause café

Session IV : Chairman : Dietmar Pfeifer

16.00 - 16.25 : Madan L. Puri (Indiana University Bloomington)
   *Convergence and Remainder Term in Multilinear Rank Statistics*

16.25 - 17.10 : Conférence de clôture Paul Deheuvels (Université Paris VI)

18.00 : Buffet de clôture en Tour Centrale de Jussieu
An Overview and Open Research Topics in Statistics of Univariate Extremes

Jan Beirlant
Katholieke Universiteit Leuven, Belgium
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This talk focuses on statistical issues arising in modeling univariate extremes of a random sample. In the last three decades there has been a shift from the area of parametric statistics of extremes, based on probabilistic asymptotic results in extreme value theory, towards a semi-parametric approach, where the estimation of the right and/or left tail-weight is performed under a quite general framework. But new parametric models can still be of high interest for the analysis of extreme events, if associated with appropriate statistical inference methodologies. After a brief reference to Gumbel’s classical block methodology and later improvements in the parametric framework, we present an overview of the developments on the estimation of parameters of extreme events and testing of extreme value conditions under a semi-parametric framework, and discuss a few challenging open research topics. This work is in collaboration with Frederico Caeiro and Ivette Gomes.
Weakly dependent processes in analysis

István Berkes
Technische Universität Graz, Austria

The theory of weak dependence starts with the investigations of Gauss (1812) on continued fractions and until the 1950’s the theory dealt almost exclusively with weak dependence phenomena in analysis and number theory. Starting with the seminal papers of Rosenblatt (1956) and Ibragimov (1962), the theory turned into a purely stochastic direction and today we have a wide and nearly complete theory giving a satisfactory description of the asymptotic properties of weakly dependent systems. The theory, however, does not cover the analytic and arithmetic applications it was motivated by and concrete function systems in analysis exhibit weak dependence phenomena not explained by the general theory. In our talk we discuss such phenomena, in particular the fascinating probabilistic properties of the system \{n\alpha\} and its empirical process and the convergence of sums $\sum c_k f(kx)$ for periodic measurable functions $f$. 
Limit Theorems for Spatial Quantiles

Philippe Berthet
Université Paul Sabatier - Toulouse 3

We will first review several related works in progress on trimming measures, spatial increments, set valued statistics, mode estimators all dealing with the geometrical localization of probability mass in space and the estimation of this localization using the empirical measure, both in infinite and finite dimension. Then we shall focus on the specific question of $d$-dimensional spatial quantiles indexed by some origin $0$ and all direction $u$, in the J.W. Tukey sense, that is using hyperplanes. The $u$-directional quantile of mass $p$ from $0$ is a point in the line $(0, u)$ where the orthogonal halfspace has probability $p$. Quantile surfaces can be drawn for any fixed $p$, and they do depend on $0$ whereas the thus described $p$-quantile halfplanes do not. Under mild regularity assumptions we obtain almost sure uniform - in $u$ and $p$ - consistency and a uniform central limit theorem for the quantile locations. Fixing $p$ implies D. Nolan results. Moreover a strong approximation of the spatial empirical quantile process is obtained by using P. Berthet and D. Mason rates for VC-classes. The probabilistic control of the key deviation events rely on exit times of a collection of correlated univariate Brownian bridges. This is where we find connections with the work of P. Deheuvels on the univariate quantile process and Bahadur-Kiefer representation: we exhibit a trajectorial explanation of the $1/n^{1/4}$ difference between Gaussian approximations of the empirical process and the quantile process. Extensions involve the replacement of halfspaces by other directionnaly ordered sets, and perspectives include the depth empirical process. This work in joint with T. Klein and A. Ahidar.
Bayesian prediction for stochastic processes

Denis Bosq
LSTA, Université Pierre et Marie Curie-Paris 6

In this presentation, after some recalls concerning general prediction theory, we adopt a Bayesian point of view for predicting real and high dimensional stochastic processes. We give two equivalent definition of a Bayesian predictor and study some properties: admissibility, prediction sufficiency, unbiasedness, comparison with efficient predictors. In the real case we apply the results to prediction of Poisson processes. We also consider prediction of the Ornstein-Uhlenbeck process in the continuous and sampled situations. Some simulations illustrate the results. Finally, in high dimension, we deal with infinite dimensional Poisson processes and autoregressive processes in Hilbert spaces.
From Boundary Crossing of Non-Random Functions to Boundary Crossing of Stochastic Processes

Victor H. de la Peña
Columbia University

In this talk I will introduce moment bounds for first crossing times that can be thought of as natural extensions of the concept of boundary crossing of non-random functions to the case of arbitrary stochastic processes. The results were inspired in part by the development of decoupling inequalities for randomly stopped processes with independent increments.
Joint work with Mark Brown and Tony Sit
Bridging centrality and extremity: refining empirical data depth using extreme value theory

John H.J. Einmahl
Tilburg University, the Netherlands

A data depth is a measure of centrality of a point with respect to a given distribution or data set. It provides a natural center-outward ordering of multivariate data points and yields systematic nonparametric multivariate analyses. In particular, the approaches derived from geometric depths (e.g. halfspace depth and simplicial depth) are especially useful since they generally reflect accurately the true probabilistic geometry underlying the data. However, the empirical geometric depths are defined to be zero outside the convex hull of the data set. This property has restricted much the utility of depth approaches in applications where the extreme outlying probability mass may be the focal point, such as in problems of classification or quality control charts with small false alarm rates.

To overcome this shortcoming, we propose to apply extreme value theory to refine the empirical estimator of half-space depth for points in the tails of the data set. This proposal provides an important linkage between data depth, which is useful for inference on centrality, and extreme value theory, which is useful for inference on extremity. The refined estimator of the half-space depth can thus extend depth utilities beyond the data hull and broaden greatly the applicability of data depth. We show that the refined depth is uniformly “ratio-consistent” on a very large region and that it substantially improves upon the original empirical estimator. This can be immediately translated into improvement for inference approaches based on depth. In a detailed simulation study we show the improvement of the estimator and how it leads to much better performance of depth applications, especially in the directions of multivariate classification and construction of nonparametric control charts.

This is joint work with Jun Li (UC Riverside) and Regina Y. Liu (Rutgers University).
Strassen type LIL results for partial sum processes in Euclidean space

Uwe Einmahl
Vrije Universiteit Brussel, Belgium

Let $X, X_1, X_2, \ldots$ be i.i.d. $d$-dimensional random vectors and as usual let $S_n := \sum_{j=1}^n X_j, n \geq 1$. Denote the Euclidean norm on $\mathbb{R}^d$ by $| \cdot |$.

Assuming $\mathbb{E}|X|^2 < \infty$ and $\mathbb{E}X = 0$, it follows from the $d$-dimensional version of the Hartman-Wintner LIL that with probability one,

$$\limsup_{n \to \infty} \frac{|S_n|}{\sqrt{2n \log \log n}} = \sigma,$$

where $\sigma^2$ is the largest eigenvalue of the covariance matrix of $X$.

There is also a $d$-dimensional version of Strassen’s LIL where one looks at the partial sum process sequence in $C_d[0,1]$ which is given by

$$S_n(t) = S_{[nt]} + (nt - [nt])X_{[nt]+1}, 0 \leq t \leq 1.$$

Then it follows that with probability one,

$$\{S_n/\sqrt{2n \log \log n} : n \geq 3\} \text{ is relatively compact in } C_d[0,1] \text{ and } C(\{S_n/\sqrt{2n \log \log n} : n \geq 3\} = \Sigma K_d$$

where $C(\{x_n\})$ denotes the cluster set of a sequence $x_n$ in a topological space and where $\Sigma$ is the symmetric, positive definite matrix satisfying $\Sigma^2 = \text{covariance matrix of } X$. The function class $K_d$ is defined by

$$K_d = \{(g_1, \ldots, g_d) : g_i(u) = \int_0^u f_i(t)dt, u \in [0,1], 1 \leq i \leq d \text{ and } \sum_{i=1}^d \|f_i\|^2_2 \leq 1\}$$

It is easy to see that this improves the above “classical” LIL result in that we also can specify the cluster set $C(\{S_n/\sqrt{2 \log \log n}\}$ which is equal to the ellipsoid $\Sigma U$ where $U$ is the Euclidean unit ball.

It is natural now to ask whether there are also analogous results for random vectors with infinite second moments. In this case one has to use different normalizing sequences $c_n$ and it is known that $C(\{S_n/c_n\})$ can be equal to any star-shaped and symmetric non-empty compact subset of $\mathbb{R}^d$.

The purpose of this talk is to give a survey on some recent work on the functional cluster set problem if $\mathbb{E}|X|^2 = \infty$. Among other things, we are able to determine all possible cluster sets in the independent component case. In the general case we can identify minimal and maximal sets for the functional cluster sets in terms of the cluster sets $C(\{S_n/c_n\})$. 

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Reduced-biased estimators of the Conditional Tail Expectation for heavy-tailed distributions

El Hadji Deme¹, Stéphane Girard², Armelle Guillou³
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Several risk measures have been proposed in the literature. In this talk, we focus on the estimation of the Conditional Tail Expectation (CTE). The asymptotic normality of estimators of the CTE has been established in the literature under a finite second moment condition. If this condition is not satisfied, no result is available. This talk deals with the estimation problem of the CTE within the class of heavy-tailed distributions, which are quite common in practice, in particular in an actuarial context. More precisely, we introduce a reduced-biased estimator and establish its asymptotic normality under very general assumptions. Our proofs are mainly based on empirical processes arguments combined with some extreme value results. Finally, we illustrate the efficiency of our estimators on a small simulation study.
Asymptotics of Bayesian variable selection procedures in linear regression

F. Javier Girón, Elías Moreno y M. Lina Martínez
University of Málaga, Spain

Variable selection is one important theoretical and practical statistical problem as can be gathered from the copious literature on this subject both from frequentist and Bayesian viewpoints. However, the performance of the various procedures which have been proposed for small, medium and large samples sizes have received less attention than it deserves. In this talk, we address the problem of comparing the performance of three well established Bayesian procedures, making special emphasis on their asymptotic behavior when the number of covariates increases as the sample size increases. In this case, the consistency of the Bayesian procedure depends not only on the priors on the regression coefficients but also on the prior used on the discrete space of all possible models, and the order of magnitude of the number of covariates. The results of our analysis show that, a combination of intrinsic priors for the parameters and the hierarchical uniform distribution on the models, asymptotically outperforms the other alternatives, and also shows a better balance of the Type I and Type II errors for small and medium size sample sizes.
The sequential empirical process for centered random variables

Erich Haeusler and Stefan Horni
Universität Giessen, Germany

If $F$ is a completely unknown distribution function on the real line, then the empirical distribution function $F_n$ based on a sample of size $n$ is the nonparametric maximum likelihood estimator for $F$, and therefore is the estimator of choice. Suppose now that some auxiliary nonparametric information about $F$ is available, like knowing the mean of $F$. Obviously, an estimator for $F$ should reflect this knowledge appropriately. One such estimator $\hat{F}_n$ can be constructed via the empirical likelihood principle; see Owen (1990), and Qin and Lawless (1994). Whereas $F_n$ puts probability mass $1/n$ on each of the observations, the estimator $\hat{F}_n$ distributes random masses on the observations in such a way that $\hat{F}_n$ does have the property known about $F$, i.e., if $F$ is known to have mean zero, $\hat{F}_n$ has mean zero, too. We consider the sequential empirical process pertaining to $\hat{F}_n$ if $F$ is known to have mean zero. It is well known that sequential empirical processes can be used to detect a distributional change point within a sequence $X_1,\ldots,X_n$ of observations. For centered distributions it is quite plausible that the version pertaining to $\hat{F}_n$ should be more effective than the classical one pertaining to $F_n$. In itself, this may not be too exciting for independent observations which are rarely known to be centered. In many models of time series analysis, however, the assumption that the distribution of the noise has mean zero is part of the model. We will show that also in such models, if distributional changes in the noise are to be detected, the sequential empirical process pertaining to $\hat{F}_n$, but now constructed from residuals, is preferable to the classical sequential empirical process of the residuals.

References

Signal Detection in High Dimension: testing sphericity against spiked alternatives

Marc Hallin
ECARES, Université libre de Bruxelles
and
ORFE, Princeton University

We consider the problem of testing the null hypothesis of sphericity for a high-dimensional covariance matrix against the alternative of a finite (unspecified) number of symmetry-breaking directions (multi-spiked alternatives). Simple analytical expressions are derived for the asymptotic power envelope and the asymptotic powers of existing tests. These asymptotic powers are shown to lie very substantially below the power envelope. In contrast, the asymptotic power of the likelihood ratio test is shown to be uniformly close to the same. Based on joint work with Marcelo Moreira, Fundação Getulio Vargas, Rio de Janeiro, and Alexei Onatski, University of Cambridge.
Approximation of Processes and Operators of Variable Non-Smoothness

Mikhail A. Lifshits
Saint Petersburg State University and Linköping University

Probabilistic models based on locally stationary processes with variable non-smoothness became recently an object of interest and a convenient tool for applications in different areas due to their flexibility. The most known representative of this class is a multifractional Brownian motion.

In the first part of the talk we build order-optimal approximations of rank $n \to \infty$ to a process with variable non-smoothness, assuming that the non-smoothness function has a unique minimum with power-type asymptotics, [1]. Small deviation probabilities of such processes will be also discussed.

In the second part we study related Riemann-Liouville operators of variable order

$$R^{\alpha(t)} f(t) := \frac{1}{\Gamma(\alpha(t))} \int_0^t f(s)(t-s)^{\alpha(t)-1} \, ds$$

acting in $L^p$-spaces and investigate their entropy numbers, [2].

References


Strong approximations to a quantile process based on independent fractional Brownian motions

David M. Mason
University of Delaware

This talk is based on joint work with Péter Kevei.

A significant amount of Paul Deheuvels’ research, as well as my collaborations with him, concerns the study of properties of the quantile process and its applications. My talk will discuss central limit theorems and strong approximations to a quantile process based upon a time dependent empirical process.

Swanson (2007) using classical weak convergence theory proved that an appropriately scaled median of \( n \) independent Brownian motions converges weakly to a mean zero Gaussian process. More recently Kuelbs and Zinn (2013) have obtained central limit theorems for a time dependent quantile process based on \( n \) independent copies of certain random processes. These include fractional Brownian motions, which may be zero or perturbed to be not zero with probability 1 at zero. Their approach is based on an extension of a result of Vervaat (1972) on the weak convergence of inverse processes.

We shall define a time dependent empirical process based on \( n \) independent fractional Brownian motions and discuss strong approximations to it by Gaussian processes. We follow the basic methodology outlined in Berthet and Mason (2006) to obtain our approximations. Surprisingly, they are in force on sequences of intervals for which weak convergence cannot hold in the limit. They lead to strong approximations and functional laws of the iterated logarithm for the quantile or inverse of these empirical processes and are obtained via Bahadur-Kiefer representations. My talk is based on joint work with Péter Kevei.
Limit Theorems for Perimeters and Areas of Random Polygons

E.V. Koroleva, Ya.Yu. Nikitin
Saint Petersburg State University

Recently W. Lao and M. Mayer (J.Mult. Anal., 99(2008), 2039 – 2052) considered $U$-max-statistics, where instead of the sum appears the maximum over the same set of indices. Such statistics often appear in stochastic geometry. Their limit distribution is related to the distribution of extreme values. In particular, Lao and Mayer obtained the limit distributions for the maximal perimeter and for the maximal area of random triangles inscribed in a circumference.

We generalize their results to convex $m$-polygons, $m \geq 3$, with random vertices on the circumference and describe the limiting Weibull distributions. Similar problem is solved for the minimal perimeter and the minimal area of circumscribed $m$-polygons. We give also the estimates for the rate of convergence. Finally, we discuss and compare the obtained results when $m \to \infty$. 
Recent contributions to the theory of chi-squared tests

V. Bagdonavicius, Mikhaïl Nikulin and Q. X., Tran
Vilnius University, Lithuania
Université Victor Segalen, Bordeaux II

The famous chi-square test of Pearson is well known, but different modifications of this test are not so well known. The theory of the chi-squared tests is developing very actively now, especially in survival analysis, accelerated trials and economy in the presence of censoring mechanism and the time depending covariates. We shall discuss here some applications of the theory of chi-squared tests in reliability and survival analysis for parametric regression models when data are right censored.

Keywords: Censored data, Chi-squared test, Complete data, Composite hypothesis, Exponential distribution, Fisher’s information, Goodness-of-fit, Maximum likelihood estimator, Parametric hypothesis, Reliability, Right censored data, Simple sample, Survival analysis, Weibull distribution.

References

Semigroups and Probability: From representation theorems to Poisson approximation

Dietmar Pfeifer
Carl von Ossietzky Universität Oldenburg, Germany

This talk is devoted to the intimate relationship between the functional analytic aspects of operator semigroups and their representations in the sense of Hille and Chernoff, and stochastic limit theorems in various probability metrics, in particular w.r.t. Poisson and Pseudo-Poisson approximation.
Convergence and Remainder Term in Multilinear Rank Statistics

Madan L. Puri
Indiana University Bloomington

For the multivariate linear rank statistics, the Berry-Esseen and the Prohorov distance estimates for the remainder term in the convergence to normality are derived. The results obtained are the extensions and generalizations of the results of Jurečková and Puri (1975, Ann. Probab.) and Bergström and Puri (1977, Ann. Statist.). The work is in collaboration with Manfred Denker and Uwe Rösler.
Bayesian Reconstruction of Past Populations for Developing and Developed Countries

Adrian E. Raftery
University of Washington

I will describe Bayesian population reconstruction, a new method for estimating past populations by age and sex, with fully probabilistic statements of uncertainty. It simultaneously estimates age-specific population counts, vital rates and net migration from fragmentary data while formally accounting for measurement error. As inputs, it takes initial bias-corrected estimates of age-specific population counts, vital rates and net migration. The output is a joint posterior probability distribution which yields fully probabilistic interval estimates of past vital rates and population numbers by age and sex. It is designed for the kind of data commonly collected in demographic surveys and censuses and can be applied to countries with widely varying levels of data quality. This is joint work with Mark Wheldon, Patrick Gerland and Samuel Clark.
Asymptotic Expansions of Distributions of Multivariate Maxima

M. Frick and R.D. Reiss
University of Siegen

This talk concerns asymptotic results for multivariate maxima within the framework of a triangular array of random vectors. Basic for our considerations is a spectral representation of the underlying distributions by means of radial and angular components and an relationship between the spectral densities and the Pickands dependence function. With respect to limiting distributions we refer to the article by M. Frick and R.-D. Reiss (2010), Limiting distributions of maxima under triangular schemes, J. Mult. Analysis 101 2346-2357. These results are supplemented by asymptotic expansions where the leading term concerns the case of asymptotic independence.
Error distributions for random grid approximations of multidimensional stochastic integrals

Carl Lindberg and Holger Rootzén
Chalmers University of Technology, Göteborg, Sweden

This talk is about joint convergence of the approximation error for several stochastic integrals with respect to local Brownian semimartingales, for nonequidistant and random grids. The conditions needed for convergence are that the Lebesgue integrals of the integrands tend uniformly to zero and that the squared variation and covariation processes converge. We also provide tools which simplify checking these conditions and which extend the range for the results. The results are used to prove an explicit limit theorem for random grid approximations of integrals based on solutions of multidimensional SDEs, and to find ways to “design” and optimize the distribution of the approximation error. As examples, strategies for discrete option hedging are briefly discussed.
Erdős-Rényi-Shepp Type Laws and Some Related Phenomena

Josef G. Steinebach
Universität zu Köln, Germany

In this review talk, we discuss some developments over the past 25 years in the area of Erdős-Rényi-Shepp type strong laws of large numbers and some related topics. Naturally, our point of view shall be a rather subjective and personal one, focussing on various significant contributions of Paul Deheuvels to this area and some of our joint works over many years. Topics to be covered among others include Erdős-Rényi type laws for renewal processes and moving quantiles, convergence rates, extended Erdős-Rényi laws and a related conjecture of Révész, Erdős-Rényi type laws for random fields, and functional versions of Erdős-Rényi laws, e.g., for partial sum processes or processes with independent increments. Moreover, we shall also discuss some joint work in progress concerning functional Erdős-Rényi laws for renewal processes.