We will consider a variant of the classical Cramér-von Mises statistic

\[ \omega^2_n = n \int_0^1 \psi^2(t)(F_n(t) - t)^2 dt. \]

There \( \psi(t) \) is the power function and \( F_n(t) \) is the empirical process. This statistic is designed to test the distribution uniformity of the observed random variable \( X \) with using the sample \( X_1, \ldots, X_n \). The examples of such statistics are the Anderson-Darling statistics with the weight function \( \psi(t) = 1/\sqrt{t(1-t)} \) and the statistics with the \( \psi(t) = t^\alpha \), considered in [2]. Let \( \xi_n(t) = \psi(t)(F_n(t) - t) \) with zero mean and the covariance function \( K_0(t, \tau) \). Let \( \lambda_i \) and \( \varphi_i(t) \), \( i = 1, 2, \ldots \) are eigenvalues and eigenfunctions of the linear operator with the kernel \( K_0(t, \tau) \). Analogously to the Durbin-Knott development we can represent the Cramér-von Mises statistic in the form

\[ n \int_0^1 \psi^2(t)(F_n(t) - t)^2 dt = \sum_{i=1}^\infty \frac{D_{i,n}^2}{\lambda_i}, \]

where

\[ D_{i,n} = \sqrt{\lambda_i} \int_0^1 \xi_n(t) \varphi_i(t) dt \]

are the normed Durbin-Knott components of the Cramér-von Mises statistic (see [3] and [4]). If \( n \to \infty \), then the limit components \( D_i \) have the standard normal distribution \( N(0, 1) \). At first, we will consider a follow statistic

\[ \omega^2_n(m) = n \int_0^1 \psi^2(t)(F_n(t) - t)^2 dt - \frac{D_{m,n}^2}{\lambda_m} = \sum_{i=1, i \neq m}^{\infty} \frac{D_{i,n}^2}{\lambda_i}. \]

This statistic has the limit

\[ \omega^2(m) = \int_0^1 \left( \xi(t) - \frac{D_m}{\sqrt{\lambda_m}} \varphi_m(t) \right)^2 dt = \sum_{i=1, i \neq m}^{\infty} \frac{D_{i}^2}{\lambda_i}. \]

Let the distribution of the \( X \) be \( F_{n,m}(t) = t + \theta \varphi_m(t)/\sqrt{n} \) for any \( m \). Distribution of \( \omega^2_n(m) \) does not depend asymptotically from parameter \( \theta \). Hence, the statistics \( \omega^2_n(m) \) can be used for testing the "contiguous" hypothesis \( H_{0,m} : F(t) = F_{n,m}(t) \). This method can be generalized to testing the hypothesis

\[ H_1 : F(t) = F_k(t) = x + [\theta_1 s_1(t) + \ldots + \theta_k s_k(t)]/\sqrt{n}, \quad (\theta_1, \ldots, \theta_k) \in \Theta, \quad t \in [0, 1]. \]

There, \( s_1(t), \ldots, s_k(t) \) are the arbitrary functions such that the functions \( F_k(t) \) are the distribution functions for all \( (\theta_1, \ldots, \theta_k) \) \( \Theta \). The number \( k \) can be equal to \( \infty \). Considered tests do not depend of parameters asymptotically and consequently it
does not need to provide its estimation. The limit distribution of the statistic can be represented by the distribution of the quadratic form of the independent or dependent normal variables. The methods of computing these distribution can be find in [1]. It was considered also a variants of the Cramér-von Mises test based on the Durbin-Knott components. We consider now the testing of standard hypothesis

\[ H_0 : F(t) = t, \quad t \in [0,1]. \]

Let normed components \( D_{i,n} \) correspond to an arbitrary \( \psi(t) \). Let \( d_1, d_2, ... \) be the sequence of the numbers such, that \( 1/d_1 + 1/d_2 + ... < \infty \). Then the statistic

\[ \sum_{i=1}^{\infty} \frac{D_{i,n}^2}{d_i} \]

is the double weighted statistic. First weighting realizes with the function \( \psi(t) \) and another weighting realizes with the sequence \( d_1, d_2, ... \). We have there the amplitude and frequency weightings.

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