Longitudinal Mixed-Membership Models for Survey Data on Disability

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Carnegie Mellon University

Seminaire Europeen: Mathematical Methods for Survival Analysis, Reliability and Quality of Life
Biostatistics Meeting
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November 22, 2010

Joint work with Daniel Manrique-Vallier, Duke University
The number and proportion of older Americans is increasing rapidly. Current generations of seniors are living longer than in previous generations.
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What do we know about the health of older Americans as they age?

- Are disabilities compressed toward the end of life?
Motivation—Disability and Long-Term Care in the U.S.

- The number and proportion of older Americans is increasing rapidly. Current generations of seniors are living longer than in previous generations.
- The elderly often need long-term care, especially in the presence of disabilities.
- What do we know about the health of older Americans as they age?
  - Are disabilities compressed toward the end of life?
  - ... or is it a slow process, spread over several years?
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- Are disabilities compressed toward the end of life?
- ... or is it a slow process, spread over several years?
- Is the process different for younger generations than for older ones?
- How is the disability in the population changing?
The number and proportion of older Americans is increasing rapidly. Current generations of seniors are living longer than in previous generations. The elderly often need long-term care, especially in the presence of disabilities. What do we know about the health of older Americans as they age? Are disabilities compressed toward the end of life? ... or is it a slow process, spread over several years? Is the process different for younger generations than for older ones? How is the disability in the population changing?

Answers to these questions require a longitudinal view that also takes into account the heterogeneity of the population.
1. Our Data—The NLTCS
2. Proposed Approach—Trajectory GoM models
   1. General Construction
   2. Basic Model
   3. Estimation
3. Example of Computations
4. Extensions (if time permits)
Longitudinal survey of people aged 65+
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Assess chronic disability
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- Assess chronic disability
- Measures ADLs and IADLs:
  - Activities of daily living (ADL): Basic self-care (eating, bathing, etc.)—6 binary measures.
  - Instrumental Activities of Daily Living (IADL): Related to independent living within a community (preparing meals, maintaining finances, etc.)—10 binary measures.
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- Each individual that enters the survey is reinterviewed in all subsequent waves until death.
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Approx. 20k individuals per wave. 45,009 unique individuals sampled in all six waves together. Each wave incorporates ≈ 5k new subjects to replace those who have died.
Sequential measurements on the same individuals allow to assess *individual* disability trajectories over time.
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Specifically, we want to

- Understand evolution over time:
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- Identify ‘typical’ evolutions over time
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Specifically, we want to

- Understand evolution over time:
  - Individuals
  - Population
- Identify ‘typical’ evolutions over time
- Account for and understand individual variability
Data—NLTCS Longitudinal View and Notation

\[ i = 1 \]

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\begin{array}{c|cccc}
 & t = 1 & t = 2 & t = 3 & t = 4 \\
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j = 1 & 0 & 0 & 1 & 1 \\
j = 2 & 0 & 0 & 0 & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
j = J & 0 & 0 & 0 & 0 \\
\hline
Age & 67 & 69 & 74 & 79 \\
Sex & F & \end{array}
\]

\[ N \text{ individuals indexed by } i \in \{1, 2, \ldots, N\} \]

\[ i = N \]

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\begin{array}{c|cccc}
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j = 1 & 0 & 0 & 1 & - \\
j = 2 & 0 & 1 & 1 & - \\
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j = J & 1 & 1 & 1 & - \\
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Age & 80 & 82 & 87 & - \\
Sex & M & \end{array}
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- Longitudinal/Cross-Sectional

Other information:
- Time dependent (e.g. Age)
- Fixed (e.g. DOB, Sex)
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- Other information
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  - Time dependent (e.g. Age)
  - Fixed (e.g. DOB, Sex)
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Clearly standard log-linear models and other standard contingency table methods won’t be of a lot of use.
Proposed Approach

- Combine the Grade of Membership Model (Woodbury et al. 1978, Erosheva et al. 2007) and the Multivariate Latent Trajectories Model (Connor, 2006).
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  - "Soft classification". Individuals "belong" to more than one class simultaneously. Individual degrees of membership. Acknowledges the fact that real individuals have unique trajectories.
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The specific longitudinal models we describe have potential application to the study of other phenomena measured at discrete points in time.
Mixed-Membership Models for PNAS Topic Modeling

5 years of articles from the *Proceeding of the National Academy of Sciences*, 1997–2001.

We analyze words in the abstracts and references, using “bag of words” and ‘bag of references” model and various choices of $K$, the number of “latent” topics.
Sampson studied 18 noviates in a monastery in the 1960s over a 4 year period and looked at friendship and other relationships.
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Each relationship at each time point produces an $18 \times 18$ adjacency matrix, of 0s and 1s.
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Each relationship at each time point produces an $18 \times 18$ adjacency matrix, of 0s and 1s.

We used a mixed-membership stochastic blockmodel to re-examine the data for friendship at 3 points in time using $K = 3$ blocks.
Assume the existence of $K$ “ideal classes” or “extreme profiles”

Assign each individual a Membership Vector:

$$g_i = (g_{i1}, g_{i2}, \ldots, g_{iK})$$

with $g_{ik} > 0$ and $\sum_{k=1}^{K} g_{ik} = 1$ ($g_i \in \Delta_{K-1}$).

For the “ideal” individuals, specify the marginal distribution of response $j$, at measurement time $t$, as a function of some time-dependent covariates.

$$\Pr(Y_{ijt} = y_{ijt} | g_{ik} = 1, X_i, \theta) = f_{\theta_{jk}}(y_{ijt} | X_{it})$$
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$$\Pr ( Y_{ijt} = y_{ijt} \mid g_{ik} = 1, X_i, \theta ) = f_{\theta_{j\mid k}} ( y_{ijt} \mid X_{it} )$$
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Mixed Membership: For a generic individual $i$, we model

$$\Pr(Y_{ijt} = y_{ijt} \mid g_i, X_i, \theta) = \sum_{k=1}^{K} g_{ik} f_{\theta_{j|k}}(y_{ijt} \mid X_{it})$$

Assuming conditional independence,

$$\Pr(Y_i = y_i \mid g_i, X_i, \theta) = \prod_{j=1}^{J} \prod_{t=1}^{N_i} \sum_{k=1}^{K} g_{ik} f_{\theta_{j|k}}(y_{ijt} \mid X_{it})$$

Assume that the membership vectors are an iid sample from a common distribution with support on the $K - 1$ dimensional unit simplex ($\Delta_{K-1}$):

$$g_i \mid \alpha \overset{iid}{\sim} G_{\alpha}$$
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For each extreme profile \((g_k = 1)\) specify trajectories of probability of disability in ADLs as a monotone function of Age:

\[
y_{ijt} \sim \text{Bernoulli} \left[ \lambda_{j|k}(\text{Age}_{it}) \right]
\]

\[
\lambda_{j|k}(X_{it}) = \logit^{-1} \left[ \beta_{0j|k} + \beta_{1j|k} \times \text{Age}_{it} \right]
\]

(Connor, 2006)
Basic Model—Distribution for \( g_i (\sim G_\alpha) \)

- Membership vectors from a Dirichlet distribution

\[
g_i \overset{iid}{\sim} \text{Dirichlet}(\alpha_0 \times \xi)
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with \( \alpha_0 > 0 \) and \( \xi = (\xi_1, \xi_2, \ldots, \xi_K) \in \Delta_{K-1} \).
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  - Dirichlet($\alpha_0 \times \xi$) with $\alpha_0 = K$ and $\xi = \left(\frac{1}{K}, \frac{1}{K}, \ldots, \frac{1}{K}\right)$ defines a uniform distribution over $\Delta_{K-1}$.
Basic Model—Distribution for $g_i$ ($\sim G_\alpha$)

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$$g_i \overset{iid}{\sim} \text{Dirichlet}(\alpha_0 \times \xi)$$

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For the membership distribution

\[ g_i \overset{iid}{\sim} \text{Dirichlet}(\alpha) \]

We use the same priors as Erosheva (2002):

\[ \alpha = \alpha_0 \times \xi \]
\[ \alpha_0 \sim \text{Gamma}(1, 5) \]
\[ \xi \sim \text{Uniform} [\Delta_{K-1}] \]
Basic Model—Priors

- For the membership distribution

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We use the same priors as Erosheva (2002):

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\[ \alpha_0 \sim \text{Gamma}(1, 5) \]

\[ \xi \sim \text{Uniform}[\Delta_{K-1}] \]

- And complete the specification with

\[ \beta_{0j|k} \overset{iid}{\sim} N(0, 100) \]

\[ \beta_{1j|k} \overset{iid}{\sim} N(0, 100) \]
Estimation—MCMC sampling

- MCMC algorithm based on a method from Erosheva (2002) for fitting GoM model to cross sectional data. Using an equivalent latent class representation for the GoM model.
- Difficult to run
  - Huge latent space.
  - Nonstandard distributions.
  - Numerical problems.
- 40,000 long chains (using the “improved” algorithm). 5 ∼ 7h runs
Test Computations—Data

Tested for six ADLs:

<table>
<thead>
<tr>
<th>ADL (j-index)</th>
<th>Abbrev</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>EAT</td>
<td>Eating</td>
</tr>
<tr>
<td>2</td>
<td>BED</td>
<td>Getting in and out of bed</td>
</tr>
<tr>
<td>3</td>
<td>MOB</td>
<td>Inside mobility</td>
</tr>
<tr>
<td>4</td>
<td>DRS</td>
<td>Dressing</td>
</tr>
<tr>
<td>5</td>
<td>BTH</td>
<td>Bathing</td>
</tr>
<tr>
<td>6</td>
<td>TLT</td>
<td>Toileting</td>
</tr>
</tbody>
</table>

Data from 6 waves (1982 - 2004).

Individuals from 2004 are only those that were already in the 1999 sample.

$N \approx 40K$
### Parameter Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate (sd)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$</td>
<td>0.264 (0.00489)</td>
</tr>
</tbody>
</table>

### ADL Parameter Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>ADL($j$)</th>
<th>Estimate Extreme Profile-$k$ (sd)</th>
<th>(k = 1)</th>
<th>(k = 2)</th>
<th>(k = 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi$</td>
<td>—</td>
<td>0.645 (0.004)</td>
<td>0.252 (0.003)</td>
<td>0.104 (0.002)</td>
<td></td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>1 (EAT)</td>
<td>-8.845 (0.313)</td>
<td>-3.103 (0.057)</td>
<td>-0.066 (0.044)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 (BED)</td>
<td>-7.02 (0.144)</td>
<td>-1.739 (0.053)</td>
<td>3.581 (0.142)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 (MOB)</td>
<td>-5.339 (0.093)</td>
<td>-0.759 (0.044)</td>
<td>5.803 (0.277)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4 (DRS)</td>
<td>-7.912 (0.216)</td>
<td>-2.256 (0.051)</td>
<td>2.042 (0.082)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5 (BTH)</td>
<td>-4.458 (0.075)</td>
<td>-0.23 (0.035)</td>
<td>6.257 (0.28)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6 (LTL)</td>
<td>-6.59 (0.148)</td>
<td>-1.768 (0.047)</td>
<td>2.506 (0.098)</td>
<td></td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>1 (EAT)</td>
<td>0.357 (0.017)</td>
<td>0.347 (0.008)</td>
<td>0.105 (0.006)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 (BED)</td>
<td>0.394 (0.01)</td>
<td>0.551 (0.013)</td>
<td>0.29 (0.012)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 (MOB)</td>
<td>0.348 (0.007)</td>
<td>0.52 (0.012)</td>
<td>0.426 (0.022)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4 (DRS)</td>
<td>0.392 (0.013)</td>
<td>0.463 (0.011)</td>
<td>0.203 (0.008)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5 (BTH)</td>
<td>0.295 (0.006)</td>
<td>0.426 (0.009)</td>
<td>0.445 (0.022)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6 (LTL)</td>
<td>0.337 (0.009)</td>
<td>0.475 (0.011)</td>
<td>0.234 (0.009)</td>
<td></td>
</tr>
<tr>
<td>$Age_{1/2}$</td>
<td>1 (EAT)</td>
<td>104.82 (0.46)</td>
<td>88.945 (0.163)</td>
<td>80.641 (0.444)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 (BED)</td>
<td>97.838 (0.167)</td>
<td>83.154 (0.089)</td>
<td>67.657 (0.173)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 (MOB)</td>
<td>95.338 (0.137)</td>
<td>81.458 (0.083)</td>
<td>66.389 (0.151)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4 (DRS)</td>
<td>100.212 (0.231)</td>
<td>84.869 (0.104)</td>
<td>69.934 (0.192)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5 (BTH)</td>
<td>95.118 (0.151)</td>
<td>80.54 (0.082)</td>
<td>65.927 (0.16)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6 (LTL)</td>
<td>99.553 (0.222)</td>
<td>83.725 (0.092)</td>
<td>69.3 (0.175)</td>
<td></td>
</tr>
</tbody>
</table>
Computations - “Posterior density” for $g_i$ (K=3)
Computations - prior/posterior for $\alpha_0 (K = 3)$
Test Computations—Profiles for $K = 3$

$k = 1$

$\begin{array}{|c|c|c|c|c|}
\hline
& j=1 \text{ (Eating)} & j=2 \text{ (In/Out bed)} & j=3 \text{ (Mobility)} & j=4 \text{ (Dressing)} \\
\hline
\text{Probability} & \begin{array}{cccc}
0.0 & 0.4 & 0.8 \\
65 & 70 & 75 & 80 & 85 & 90 & 95 \\
\end{array} \\
\hline
\end{array}$

$k = 2$

$\begin{array}{|c|c|c|c|c|}
\hline
& j=1 \text{ (Eating)} & j=2 \text{ (In/Out bed)} & j=3 \text{ (Mobility)} & j=4 \text{ (Dressing)} \\
\hline
\text{Probability} & \begin{array}{cccc}
0.0 & 0.4 & 0.8 \\
65 & 70 & 75 & 80 & 85 & 90 & 95 \\
\end{array} \\
\hline
\end{array}$

$k = 3$

$\begin{array}{|c|c|c|c|c|}
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& j=1 \text{ (Eating)} & j=2 \text{ (In/Out bed)} & j=3 \text{ (Mobility)} & j=4 \text{ (Dressing)} \\
\hline
\text{Probability} & \begin{array}{cccc}
0.0 & 0.4 & 0.8 \\
65 & 70 & 75 & 80 & 85 & 90 & 95 \\
\end{array} \\
\hline
\end{array}$
Test Computations—From profiles to Individuals

\[ k = 1 \]

\[ k = 2 \]

\[ k = 3 \]
Test Computations—From profiles to Individuals

$k = 1$

$k = 2$

$k = 3$
Test Computations—From profiles to Individuals

\[ k = 1 \]

\[ j = 2 \text{ (In/Out bed)} \]

\[ j = 2 \text{ (In/Out bed)} \]

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\[ j = 2 \text{ (In/Out bed)} \]
Test Computations—Individual Trajectories

j=1 (Eating)

j=2 (In/Out bed)

j=3 (Mobility)

j=4 (Dressing)

j=5 (Bathing)

j=6 (Toileting)
Extensions

1. Modeling of generational differences through generation-dependent group membership distributions.
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   - Assess changes over time as a function of individual’s “generational group”.

2. Joint modeling of survival times and disability acquisition.

3. Other trajectory functions (e.g., step functions).
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   - Assess changes over time as a function of individual’s “generational group”.
   - Allow to answer the question: “Are younger generations acquiring disabilities in a different way than older ones?”
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1. Modeling of generational differences through generation-dependent group membership distributions.
   - Assess changes over time as a function of individual’s “generational group”.
   - Allow to answer the question: “Are younger generations acquiring disabilities in a different way than older ones?”

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3. Other trajectory functions (e.g. step functions).
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Extensions (1) - Modeling Generational differences

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- Study generations (i.e. people that were born in the same period) to understand changes in how people acquire disabilities.
- Longitudinal data allows us to compare whole aging life trajectories for different individuals from different generations.
Approach: make group membership dependent on the generation to which the individual belongs, keeping the extreme trajectories fixed:

\[
\Pr(Y_{ijt} = y_{ijt} | g_i, X_i, \theta) = \sum_{k=1}^{K} g_{ik} f_{\theta_i|k}(y_{ijt} | Age_{it})
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This way we can assess the distribution of membership scores for different generational groups.
5 (rather arbitrary) “Generational Groups”.

Born...

1...before 1906 (up to 1873)

2 between 1906 and 1914,

3 between 1914 and 1919,

4 between 1919 and 1926,

5 after 1926 (up to 1934).

Individuals form younger generations tend to be closer to “healthy” trajectory profiles.
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Other Extensions

1. Joint modeling of survival times and disability acquisition.
   - Use survival information to achieve better classification.
   - Understand the relationship between disability and mortality.

2. Other trajectory functions (e.g. step functions)
   - Test the constraints imposed by the selection of disability trajectory curves.


4. Model choice—picking the value of $K$.

5. Incorporating fuller set of covariates.

6. Adapting all of these models and methods for other surveys—e.g., Health and Retirement survey, and the new National Health and Aging Trends Study (NHATS).
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Mixed membership accounts for individual variability. In this particular application, extreme profile trajectories illustrate typical ways of aging. Mixed membership acknowledges the fact that not everybody ages the same way!

Making individual membership scores dependent on the individual's generation allows to assess changes on the ways of aging.

General methodology. Can be applied in other settings!
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Summary

- We’ve developed a method that allows understanding discrete longitudinal data from heterogeneous populations.
- Modeling a small number of extreme profiles uncovers and summarize typical progressions over time.
- Mixed membership accounts for individual variability.
- In this particular application,
  - Extreme profile trajectories illustrate typical ways of aging.
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The End